FYUG Odd Semester Exam., 2024

STATISTICS

(3rd Semester)

Course No. : STADSC-202T

(Limit Theorems and Sampling Distributions)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following : 2×2=4

(a) Define convergence in probability.

(b) State strong law of large numbers.

(c) If X_n converges to X in probability, then prove that $X_n - X$ converges to zero in probability.

2. Answer *either (a) or (b)* : 10

(a) *(i)* State and prove Chebyshev's inequality. 3

(ii) State and prove De Moivre-Laplace central limit theorem. 4

(iii) If X is a random variable such that $E(X) = 3$ and $E(X^2) = 13$, then use Chebyshev's inequality to determine a lower bound for

$$P(-2 < X < 8)$$ 3

(b) (i) State and prove Bernoulli's law of large numbers. 3

(ii) By applying the central limit theorem to a sequence of random variables with Poisson distribution, prove that

$$\operatorname*{Lt}_{n \to \infty} e^{-n} \sum_{r=0}^{n} \frac{n^r}{r!} = \frac{1}{2}$$ 4

(iii) State and prove weak law of large numbers. 3

Unit—II

3. Answer any *two* from the following : 2×2=4

(a) Define sampling distribution of a statistic.

(b) Write the probability density function (p.d.f.) and cumulative distribution function of a rth order statistic.

(c) Prove that sample proportion is an unbiased estimate of population proportion.

4. Answer *either* (a) *or* (b) : 10

(a) (i) Obtain standard error of the sample mean drawn from a finite population with SRSWR and SRSWOR. 4

(ii) Explain briefly the applications of order statistics. 3

(iii) Obtain the p.d.f. of rth order statistic. 3

(b) (i) Derive the sampling distribution of the mean of a random sample drawn from a normal population. 6

(ii) Obtain the joint p.d.f. of $X_{(r)}$ and $X_{(s)}$, $r < s$ in a random sample of size n from a population with continuous distribution function $F(x)$. Hence deduce the p.d.f. of the range $R = X_{(n)} - X_{(1)}$. 4

Unit—III

5. Answer any *two* from the following : 2×2=4

(a) Define critical region and level of significance.

(b) Define simple and composite hypotheses with examples.

(c) Write with examples one-tailed and two-tailed tests.

6. Answer *either (a) or (b)* : 10

(a) (i) Write a short note on large sample test in test of significance. Also define level of confidence. 3+1=4

(ii) Obtain the large sample test for difference of proportions of two distinct populations. 3

(iii) Explain the utility of standard error in test of significance for large samples. 3

(b) (i) Describe the test of significance for large samples to test the difference of means of two distinct populations. 3

(ii) A die is thrown 9000 times and a throw of 1 and 2 is observed 3120 times. Show that the die cannot be regarded as an unbiased one. Also find the limits between which the probability of a throw of 1 or 2 lies. 4

(iii) Describe the procedure for test of significance for difference of standard deviations of two distinct populations. 3

UNIT—IV

7. Answer any *two* from the following : $2 \times 2 = 4$

(a) Write the p.d.f. of χ^2-distribution and state the relationship between mean and variance of χ^2-distribution.

(b) State the conditions for validity of χ^2-test.

(c) If X follows χ^2-distribution with n d.f., then prove that

$$Y = \frac{1}{2} \sim \gamma\left(\frac{n}{2}\right)$$

8. Answer *either (a) or (b)* : 10

(a) (i) Define χ^2-variate with n degrees of freedom and derive its distribution. 1+3=4

(ii) If X and Y are independent χ^2-variates with n_1 and n_2 degrees of freedom, then prove that

$$U = \frac{X}{Y}$$

is a $\beta_2\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$ variate. 3

(iii) If X is a χ^2-variate with n degrees of freedom, then prove that for large n, $\sqrt{2X} \sim N(\sqrt{2n}, 1)$. 3

(b) (i) Obtain the moment generating function (m.g.f.) of χ^2-distribution and hence obtain mean and variance. 4

(ii) For the 2×2 contingency table

a	b
c	d

prove that χ^2-test of independence gives

$$\chi^2 = \frac{N(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)}$$

where $N = a+b+c+d$. 3

(iii) Obtain mode and skewness of χ^2-distribution with n degrees of freedom (d.f.). 3

UNIT—V

9. Answer any *two* from the following : 2×2=4

(a) If $t \sim t_{(n)}$, then write the mean and variance of t-distribution.

(b) Write the applications of Snedecor's F-distribution.

(c) State the assumption of Student's t-distribution.

10. Answer *either* (a) *or* (b) : 10

(a) (i) Define Fisher's t-statistic and derive its distribution. 1+3=4

(ii) Describe the t-test for single mean mentioning clearly the underlying assumptions. 3

(iii) Obtain the relation between F- and χ^2-distribution. 3

(b) (i) Define Snedecor's F-statistic and derive its distribution. 1+3=4

(ii) Derive the relation between t- and F-distribution. 3

(iii) Prove that if $n_1 = n_2$, the median of F-distribution is at $F = 1$ and that the quartiles Q_1 and Q_3 satisfy the condition $Q_1 Q_3 = 1$. 3

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